Royaume du Maroc

Ministère de l'Éducation Nationale du Préscolaire et des Sports

EXAMEN D'OBTENTION DU CERTIFICAT DU BACCALAUREAT

Série ou Option : ....

Date d'examen : ....

Matière de : الرياضيات

Nom et Signature du correcteur : ....

Note globale	
En chiffres	17,25/20
En lettres	....

Numéro d'archivage: 0982/11

NOTATION PARTIELLE

Exercise 1 :

1 We have $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{\ln(x)}{x^2-1} = \lim\limits_{x\to 1^+} \frac{\ln(x)}{x-1} \cdot \frac{1}{x+1} = \frac{1}{2}$

because : $\lim\limits_{x\to 1^+} \frac{\ln(x)}{x-1} = 1$ and $\lim\limits_{x\to 1^+} \frac{1}{x+1} = \frac{1}{2}$

Thus $\lim\limits_{x\to 1^+} f(x) = f(1) \Leftrightarrow f$ is right continuous at 1.

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2 $\lim\limits_{x\to +\infty} \frac{\ln(x)}{x^2-1} = \lim\limits_{x\to +\infty} \frac{\ln(x)}{x} \cdot \frac{x}{x^2-1} = 0$

because $\lim\limits_{x\to +\infty} \frac{\ln(x)}{x} = 0$ and $\lim\limits_{x\to +\infty} \frac{x}{x^2-1} = \lim\limits_{x\to +\infty} \frac{x}{x^2} = \lim\limits_{x\to +\infty} \frac{1}{x} = 0$

Thus $\lim\limits_{x\to +\infty} f(x) = 0 \Rightarrow (C)$ admits an horizontal tangent at $(+\infty)$ (x-axis).

0,5

3-a We have $t = (x-1)^2$ - So $\frac{-\sqrt{t} + \ln(1+\sqrt{t})}{t} = \frac{-\sqrt{(x-1)^2} + \ln(1+\sqrt{(x-1)^2})}{(x-1)^2}$

$x > 1 \Rightarrow \sqrt{(x-1)^2} = x-1$ $\quad = \frac{-(x-1) + \ln(1+x-1)}{(x-1)^2}$

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Thus $\forall x \in]1;+\infty[$: $\frac{-\sqrt{t} + \ln(1+\sqrt{t})}{t} = \frac{1-x+\ln(x)}{(x-1)^2}$

3-b We have $v : t \mapsto 1+\sqrt{t}$ is differentiable on $\mathbb{R}_+^*$

because $t > 0$

and $v(\mathbb{R}_+^*) =]1;+\infty[$ (because $\lim\limits_{t\to 0^+} v(t) = 1$

and $\lim\limits_{t\to +\infty} v(t) = +\infty$ ($\lim\limits_{t\to +\infty} \sqrt{t} = +\infty$))

$\Rightarrow v(\mathbb{R}_+^*) \subset \mathbb{R}_+^*$

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NOTATION PARTIELLE

And ln is differentiable on $\mathbb{R}_+^*$

Thus $t \mapsto \ln(1+\sqrt{t})$ is differentiable on $\mathbb{R}_+^*$

We have: $t \mapsto -\sqrt{t}$ is differentiable on $\mathbb{R}_+^*$ (Square root function)

$t \mapsto \frac{1}{t}$ is differentiable on $\mathbb{R}_+^*$ (Restriction of rational and continuous function)

We have so: $g: t \mapsto -\sqrt{t} + \ln(1+\sqrt{t})$ is differentiable on $\mathbb{R}_+^*$

$\Rightarrow g: t \mapsto -\sqrt{t} + \ln(1+\sqrt{t})$ is continuous on $\mathbb{R}_+^*$

We have $\lim_{t \to 0^+} g(t) = \lim_{t \to 0^+} -\sqrt{t} + \ln(1+\sqrt{t}) = 0 = g(0)$

$\Rightarrow g$ is right continuous at 0

$\Rightarrow g$ is differentiable on $\mathbb{R}_+^*$ and continuous on $\mathbb{R}_+$

(~~We~~ ~~$\varphi(t) =$~~) So φ is differentiable on $\mathbb{R}_+^*$ and continuous on $\mathbb{R}_+$ (Product of two differentiable and continuous functions) / $\varphi(t) = \frac{-\sqrt{t} + \ln(1+\sqrt{t})}{t}$

Thus φ is continuous on $[0;t] \subset \mathbb{R}_+$ $t>0$

φ is differentiable on $]0;t[\subset \mathbb{R}_+^*$ $t>0$

Thus According to mean value theorem.

$\exists c \in]0;t[\,/\, \frac{\varphi(t)-\varphi(0)}{t-0} = \varphi'(c)$

$\varphi'(t) = \frac{(-\sqrt{t}+\ln(1+\sqrt{t}))'}{}$

$= \frac{\left(\frac{1}{2\sqrt{t}} + \frac{\frac{1}{2\sqrt{t}}}{1+\sqrt{t}}\right)t + \sqrt{t} - \ln(1+\sqrt{t})}{t^2}$

$= \frac{\frac{t}{2\sqrt{t}}\left(\frac{1}{2(1+\sqrt{t})} - 1\right) + \sqrt{t} - \ln(1+\sqrt{t})}{t^2}$

$= \frac{\frac{\sqrt{t}}{2}\left(\frac{1}{2(\sqrt{t}+1)} - 1\right) + \sqrt{t} - \ln(1+\sqrt{t})}{t^2}$

$=$

TOTAL NOTE/PAGE

N.B : Il est interdit aux candidats de signer leur composition ou d'y mettre un signe quelconque pouvant révéler leur identité

3- a We denote $\varphi(t) = -\sqrt{t} + \ln(1+\sqrt{t})$

We have $u : t \to 1+\sqrt{t}$ is differentiable on $\mathbb{R}_+^*$

and $u(\mathbb{R}_+^*) =]1;+\infty[$ $\left(\lim_{t\to 0^+} u(t) = 1 \quad \lim_{t\to+\infty} u(t) = +\infty\right)$

and $\ln$ is differentiable on $\mathbb{R}_+^*$ and $u(\mathbb{R}_+^*) \subset \mathbb{R}_+^*$
and $t \to -\sqrt{t}$ is differentiable on $\mathbb{R}_+^*$.
Thus φ is differentiable on $\mathbb{R}_+^*$ (Composit and sum of two differentiable functions).

φ is differentiable on $\mathbb{R}_+^* \Rightarrow \varphi$ is continuous on $\mathbb{R}_+^*$

We have $\lim_{t\to 0^+} \varphi(t) = \lim_{t\to 0^+} -\sqrt{t} + \ln(1+\sqrt{t}) = 0 = \varphi(0)$

$\Rightarrow \varphi$ is continuous at right of 0

Thus φ is continuous on $\mathbb{R}_+$

We have: φ is differentiable on $\mathbb{R}_+^* \Rightarrow \varphi$ is differentiable on $]0;t[$ $t>0$.

φ is continuous on $\mathbb{R}_+ \Rightarrow \varphi$ is continuous on $[0;t]$ $t>0$.

Thus According to mean value theorem.

$\exists c \in]0;t[\;/\; \dfrac{\varphi(t)-\varphi(0)}{t-0} = \varphi'(c)$

$$\varphi'(t) = \left(-\sqrt{t} + \ln(1+\sqrt{t})\right)' = \frac{-1}{2\sqrt{t}} + \frac{\frac{1}{2\sqrt{t}}}{1+\sqrt{t}}$$

$$= \frac{1}{\sqrt{t}}\left(\frac{1}{2(1+\sqrt{t})} - \frac{1}{2}\right)$$

So $\dfrac{\varphi(t)}{t} = \dfrac{1}{\sqrt{c}}\left(\dfrac{1}{2(1+\sqrt{c})} - \dfrac{1}{2}\right)$

$\Leftrightarrow \dfrac{\varphi(t)}{t} = \dfrac{1}{\sqrt{c}}\left(\dfrac{1 - 1 - \sqrt{c}}{2(1+\sqrt{c})}\right)$

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النقطة الجزئية

مجموع نقط الصفحة

المملكة المغربية
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امتحان نيل شهادة البكالوريا

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~~...~~ Thus $\frac{\varphi(t)}{t} = \frac{-1}{2(1+\sqrt{c})}$

We have $0 < c < t$

$\Rightarrow \sqrt{c} < \sqrt{t}$ $x \mapsto \sqrt{x}$ is increasing

$\Rightarrow 1+\sqrt{c} < 1+\sqrt{t}$ $1 < 1+\sqrt{c}$

$\Rightarrow \frac{1}{1+\sqrt{t}} < \frac{1}{1+\sqrt{c}}$ $\Rightarrow \frac{1}{1+\sqrt{c}} < 1$

$\Rightarrow \frac{-1}{2}< \frac{-1}{1+\sqrt{c}}$

$\Rightarrow \frac{-1}{2(1+\sqrt{c})} < \frac{-1}{2(1+\sqrt{t})}$ $\Rightarrow \frac{-1}{2} < \frac{-1}{2(1+\sqrt{c})}$

$\Rightarrow \frac{\varphi(t)}{t} < \frac{-1}{2(1+\sqrt{t})}$ (1) $\Rightarrow \frac{-1}{2} < \frac{\varphi(t)}{t}$

Thus from (1) and (2): $\frac{-1}{2} < \frac{-\sqrt{t}+\ln(1+\sqrt{t})}{t} < \frac{-1}{2(1+\sqrt{t})}$

3-c) We have in 3-b

$\forall t > 0$ $\frac{-1}{2} < \frac{-\sqrt{t}+\ln(1+\sqrt{t})}{t} < \frac{-1}{2(1+\sqrt{t})}$

and according to 3-a $\frac{-1}{2} < \frac{1-x+\ln(x)}{(x-1)^2} < \frac{-1}{2(1+\sqrt{t})}$

We have when $x \mapsto 1^+$ $t \to 0^+$ because $t = (x-1)^2$

So $\lim_{x \mapsto 1^+} \frac{-1}{2(1+\sqrt{t})} = \lim_{t \to 0^+} \frac{-1}{2(1+\sqrt{t})} = \frac{-1}{2}$

Thus $\lim_{x \mapsto 1^+} \frac{1-x+\ln(x)}{(x-1)^2} = \frac{-1}{2}$

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NOTATION PARTIELLE

4-a We have $\forall x \in]1;+\infty[$

Let's prove : $\frac{f(x)-\frac{1}{2}}{x-1} = -\frac{\ln(x)}{x-1}\cdot\frac{1}{2(x+1)} + \frac{\ln(x)-x+1}{2(x-1)^2}$

We have : $\cancel{-\left(\frac{\ln(x)}{x-1}\cdot\frac{1}{2(x+1)} + \frac{\ln(x)-x+1}{2(x-1)^2} = \frac{-\ln(x)}{2(x^2-1)} + \frac{\ln(x)-x+1}{2(x-1)}\right)}$

$\cancel{= \frac{1}{x-1}\left(\frac{-\ln(x)}{2(x+1)} + \frac{\ln(x)-x+1}{2(x-1)}\right)}$

$\cancel{= \frac{1}{x-1}\left(\frac{1}{2}\left(-\ln(x)(x-1)+\ln(x)-x+1\right.\right.}$

$-\frac{\ln(x)}{x-1}\cdot\frac{1}{2(x+1)} + \frac{\ln(x)-x+1}{2(x-1)^2} = \frac{1}{x-1}\left(\frac{1}{2}\left(\frac{-\ln(x)}{x+1} + \frac{\ln(x)-x+1}{x-1}\right)\right)$

$= \frac{1}{x-1}\cdot\frac{1}{2}\left(\frac{-\ln(x)(x-1)+(\ln(x)-x+1)(x+1)}{(x^2-1)}\right)$

$= \frac{1}{x-1}\cdot\frac{1}{2}\left(\frac{\ln(x)(x+1-x+1)+(1-x)(x+1)}{x^2-1}\right)$

$= \frac{1}{x-1}\cdot\frac{1}{2}\left(\frac{2\ln(x)+1-x^2}{x^2-1}\right)$

Thus: $(\forall x \in]1;+\infty[)$ $= \frac{1}{x-1}\cdot\frac{1}{2}\left(2f(x)-1\right)$

$-\frac{\ln(x)}{x-1}\cdot\frac{1}{2(x+1)} + \frac{\ln(x)-x+1}{2(x-1)^2} = \frac{f(x)-\frac{1}{2}}{x-1}$.

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N.B : Il est interdit aux candidats de signer leur composition ou d'y mettre un signe quelconque pouvant révéler leur identité

NOTATION PARTIELLE

4.b We have $\lim\limits_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = \lim\limits_{x \to 1^+} \frac{f(x)-1/2}{x-1}$

(4-a)2 $= \lim\limits_{x \to 1^+} \frac{-\ln(x)}{x-1} \times \frac{1}{2(x+1)} + \frac{\ln(x)-x+1}{2(x-1)^2}$

We have $\lim\limits_{x \to 1^+} \frac{-\ln(x)}{x-1} = -1$ (usual limit)

$\lim\limits_{x \to 1^+} \frac{1}{2(x+1)} = \frac{1}{4}$ (Rational function).

$\lim\limits_{x \to 1^+} \frac{\ln(x)-x+1}{2(x-1)^2} = \frac{-1}{4}$ (q. 3-c)

Thus $\lim\limits_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = -1 \times \frac{1}{4} - \frac{1}{4} = -\frac{2}{4} = -\frac{1}{2}$

Thus f is right differentiable at 1 and $f_r'(1) = \frac{-1}{2}$

Thus (C) admits a half tangent at right of 1

of equation $T: \begin{cases} y = f_r'(1)(x-1) + \frac{1}{2} = f(1) \\ x \geq 1 \end{cases}$

$T: \begin{cases} y = -\frac{1}{2}(x-1) + \frac{1}{2} = \frac{-x+1}{2} \\ x \geq 1 \end{cases}$

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5 We have $\forall x \in [1;+\infty[\ 1 \leq t \leq x \Rightarrow$: $t^2-1 = (t-1)(t+1)$
$(t-1) \geq 0$

Thus $0 \leq t^2-1 \leq t^2-1$ and $t+1 \geq 0$

$0 \leq \frac{t^2-1}{t^3} \leq \frac{t^2-1}{t^2}$ — $\begin{cases} 1 \leq t \\ t^2 \leq t^3 \\ \frac{1}{t^3} \leq \frac{1}{t^2} \end{cases}$

Thus $\forall x \in [1;+\infty[$

Thus $0 \leq \int_1^x \frac{t^2-1}{t^3}\,dt \leq \int_1^x \frac{t^2-1}{t^2}\,dt$

integration keeps order

Thus $\forall x \in [1;+\infty[$ $\boxed{0 \leq I(x) \leq J(x)}$.

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5- b We have $\forall x \in]1; +\infty[$ $\quad I(x) = \int_1^x \frac{t^2-1}{t^3}\,dt$

$\ln'(t) = \frac{1}{t}$ $\quad = \int_1^x \frac{1}{t} - \frac{1}{t^3}\,dt$

$\left(\frac{1}{2t^2}\right)' = \frac{1}{2} \cdot \frac{2t}{t^4} = \frac{1}{t^3}$ $\quad = \left[\ln|t| + \frac{1}{2t^2}\right]_1^x$

$= \ln(x) + \frac{1}{2x^2} - \ln(1) - \frac{1}{2}$

$= \ln(x) + \frac{1-x^2}{2x^2}$

$I(x) = \ln(x) - \frac{x^2-1}{2x^2}$

$J(x) = \int_1^x \frac{t^2-1}{t^2}\,dt$

$\left(\frac{1}{t}\right)' = -\frac{1}{t^2}$ $\quad = \int_1^x 1 - \frac{1}{t^2}\,dt$

$= \left[t + \frac{1}{t}\right]_1^x$

$= x + \frac{1}{x} - 1 - 1$

$= \frac{x^2+1-2x}{x}$

Thus $J(x) = \frac{(x-1)^2}{x}$

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5-c We have $f'(x) = \left(\frac{\ln(x)}{x^2-1}\right)' = \frac{\ln'(x)(x^2-1) - (x^2-1)'\ln(x)}{(x^2-1)^2}$

$f'(x) = \frac{\frac{x^2-1}{x} - 2x\ln(x)}{(x^2-1)^2}$

$= \cancel{x}$

and $\frac{-2}{(x+1)^2} + \frac{I(x)}{J(x)} = \frac{-2}{(x+1)^2} + \frac{\ln(x) - \frac{x^2-1}{2x^2}}{\frac{(x-1)^2}{x}}$

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مجموع نقط الصفحة

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المملكة المغربية
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$\cancel{= x^2+1}$

$$\ln(x) - \frac{x^2-1}{2x^2}$$

$$= \frac{-2}{(x+1)^2} \times \frac{\ }{(x-1)^2}$$

$$= \frac{-2}{(x+1)^2} \times \frac{x\ln(x)\frac{x}{\ } - \frac{x^2-1}{2x}}{(x-1)^2}$$

$$= \frac{\frac{x^2-1}{x} \quad 2x\ln(x)}{(x+1)(x-1)(x+1)(x-1)}$$

$$f'(x) = \frac{\frac{x^2-1}{x} - 2x\ln(x)}{(x^2-1)^2}$$

(0,25)

Thus $\forall x \in]1;+\infty[$ $\boxed{f'(x) = \frac{-2}{(x+1)^2} \times \frac{J(x)}{J(x)}}$

3-d We have $I(x) \geq 0$ and $J(x) > 0$ $x > 1$

$I(1) = J(1) = 0$ Just

Thus $\frac{I(x)}{J(x)} \geq 0$ and $\frac{-2}{(x+1)^2} \leq 0$

$\Rightarrow \frac{-2}{(x+1)^2} \times \frac{I(x)}{J(x)} \leq 0 \Rightarrow f'(x) \leq 0$ ①

We have $x > 1 \Rightarrow (x+1)^2 > 4$

(0,75)

$\Rightarrow \frac{1}{(x+1)^2} < \frac{1}{4}$

$\Rightarrow \frac{-2}{(x+1)^2} < \frac{-1}{2}$

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NOTATION PARTIELLE

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We have also $I(x) \leq J(x)$ $\forall x \in]1;+\infty[$ in q. 5-a

$$\Rightarrow \frac{I(x)}{J(x)} \leq 1$$

$$\Rightarrow \frac{-2}{(x+1)^2} \times \frac{I(x)}{J(x)} \geq \frac{-1}{2}$$

So $f'(x) \geq \frac{-1}{2}$

Thus from ① and ② $\forall x > 1$ $-\frac{1}{2} \leq f'(x) \leq 0$

6-a

We have in q. 5-d $f'(x) \leq 0 \Rightarrow f$ is strictly decreasing on $]1;+\infty[$ because $f'(x) \neq 0$ $\forall x \in]1;+\infty[$ because just $I(1) = 0 = J(1)$ also $\frac{-2}{(x+1)^2} \neq 0$

So:

x	1	$+\infty$
$f'(x)$	$-$	
$f(x)$	$1/2$	$\to 0$

$\lim_{x \to +\infty} f(x) = 0$ at q. 2 and $f(1) = 1/4$.

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TOTAL NOTE/PAGE 0,5

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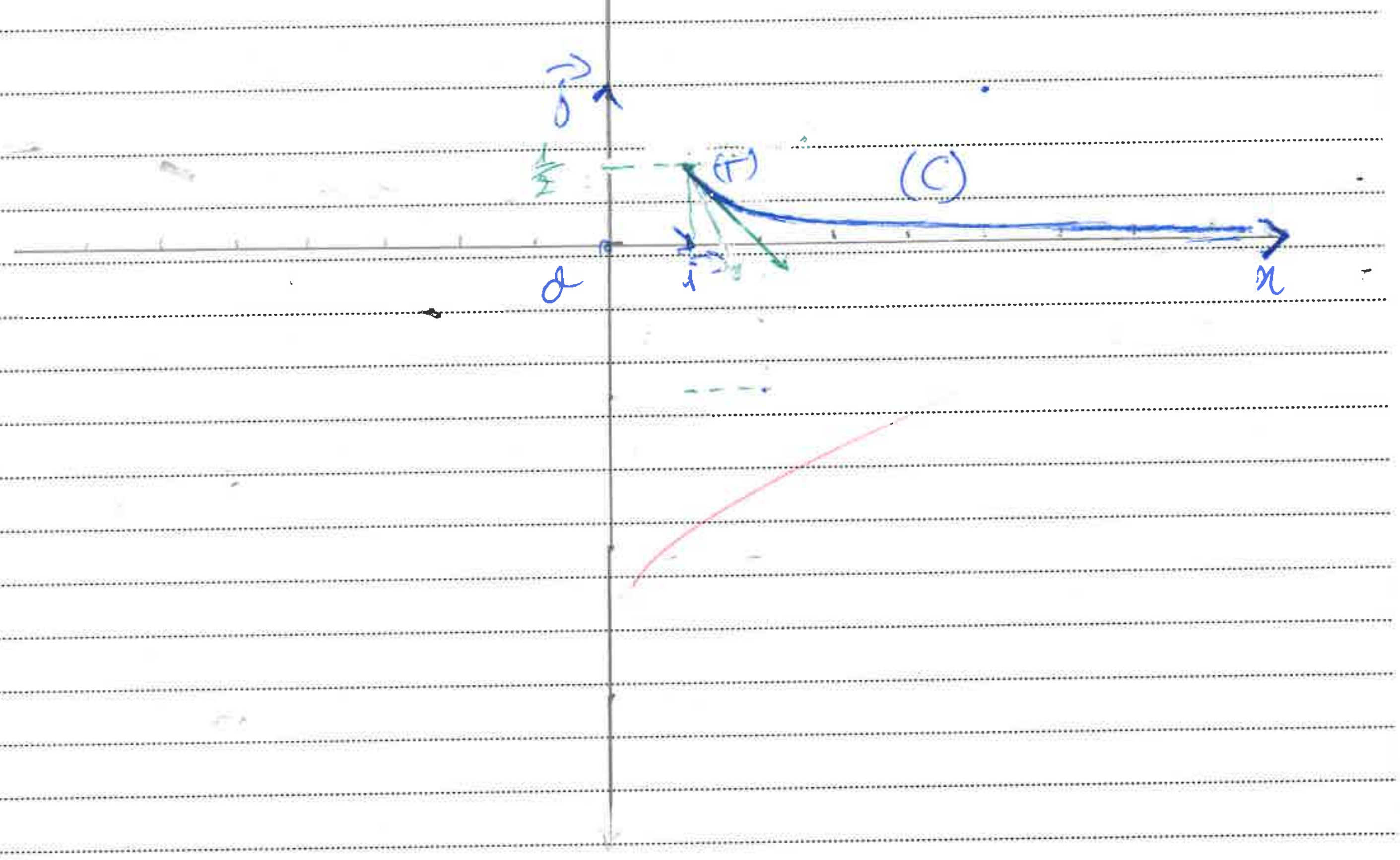
NOTATION PARTIELLE

6-b

$T : \begin{cases} y = \frac{-x}{t} \\ x \geq 0 \end{cases}$

$\lim\limits_{x \to 1^+}$



7 We denote $g(x) = f(x) + 1 \cancel{-} x$

We have f is strictly decreasing on $]1,2[\subset]1;+\infty[$

(Variation table)

and $x \mapsto -x+1$ is continuous and strictly decreasing

on $]1;2[$ (Restriction of Polynomial function) and $a = -1$

from the graph of f, f is continuous on $]1;2[$

Thus g is strictly decreasing and continuous on $]1,2[$

(Sum of 2 decreasing and continuous functions)

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Thus g is bijective from $]1,2[$ to $g(]1,2[) =]\lim\limits_{x\to 2^-} g(x); \lim\limits_{x\to 1^+} g(x)[$

$\lim\limits_{x\to 2^-} g(x) = \lim\limits_{x\to 2^-} f(x) + 1 - x = f(2) - 1$

$= \frac{\ln(2)}{3} - 1$ $\quad =]-1 + \frac{\ln(2)}{3}; \frac{1}{4}[$

$\lim\limits_{x\to 1^+} f(x) - 1 + x = f(1) = \frac{1}{4}$ / We have $2 < e$ $\overset{\ln \text{ is increasing}}{\Rightarrow} \ln(2) < 1 \Rightarrow \frac{\ln(2)}{3} < 1$

$\Rightarrow \frac{\ln(2)}{3} - 1 < 0$

Thus $0 \in g(]1;2[)$

Thus According to bijectivity Theorem the equation $g(x) = 0$ Admits a unique solution in $]1,2[$

8-a We have f is continuous on $[x;y]$ $\forall (x,y) \in$ [illegible] (graph)

and f is differentiable on $]x;y[$

and According to q.5-d $\forall x \in]1;+\infty[$ $|f'(x)| \leq \frac{1}{2}$

Thus according to Mean value theorem the equality

$\forall (x,y) \in$ $\quad |f(y) - f(x)| \leq \frac{1}{2}|x - y|$

We take $a_n = y$ and $x = a$ $\quad a_n \in [1;+\infty[$

Thus: $|f(a_n) - f(a)| \leq \frac{1}{2}|a_n - a|$

$\Rightarrow |a_n + \cancel{1} - 1 - a + \cancel{1}| \leq \frac{1}{2}|a_n - a|$

$\Rightarrow \forall n \in \mathbb{N}$ $|a_{n+1} - a| \leq \frac{1}{2}|a_n - a|$

8-b for $n = 0$ We have $|a_0 - a| \leq \left(\frac{1}{2}\right)^0 |a_0 - a|$ is correct

We suppose $|a_n - a| \leq \left(\frac{1}{2}\right)^n |a_0 - a|$ for $n \in \mathbb{N}$

We prove $|a_{n+1} - a| \leq \left(\frac{1}{2}\right)^{n+1} |a_0 - a|$

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النقطة الجزئية

We have $\lim$ q. 7-a $\forall n \in \mathbb{N}$ $|a_{n+1}-a| \leq \left(\frac{1}{2}\right)|a_n - a|$

and $|a_n - a| \leq \left(\frac{1}{2}\right)^n |a_0 - a|$

$\Rightarrow |a_{n+1} - a| \leq \left(\frac{1}{2}\right)^{n+1} |a_0 - a|$

Thus by induction $\forall n \in \mathbb{N}$ / $|a_n - a| \leq \left(\frac{1}{2}\right)^n |a_0 - a|$

0,5

7-e We have $\forall n \in \mathbb{N}$ $|a_n - a| \leq \left(\frac{1}{2}\right)^n |a_0 - a|$

and $\lim\limits_{n \to +\infty} \left(\frac{1}{2}\right)^n |a_0 - a| = 0$ bcs: $\left|\frac{1}{2}\right| < 1$

Thus $\lim\limits_{n \to +\infty} a_n = a$ $\Rightarrow (a_n)_{n \in \mathbb{N}}$ is convergent.

0,25

Exercise 2:

1-a) We have $t \to (e^t)^2$ is differentiable on $[0;1]$ (Product of two differentiable functions) $t \to e^t$ is differentiable on $\mathbb{R}$ $(e^t)^2 = e^t \times e^t$

Thus F is differentiable on $[0;1] \Rightarrow$ F is continuous on $[0;1]$

Thus $\forall x \in [0;1]$ $F'(x) = (e^x)^2$ and $\forall t \in [0;1]$ $e^t > 0$ $\Rightarrow (e^t)^2 > 0$

Thus $\forall x \in [0;1]$ $F'(x) > 0$

$\Rightarrow$ F is strictly increasing on $[0;1]$.

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1-b) We have F is strictly increasing on $[0;1]$ and continuous on $[0;1]$

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مجموع نقط الصفحة

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Numéro d'archivage

EXAMEN D'OBTENTION DU CERTIFICAT DU BACCALAUREAT

Royaume du Maroc

Ministère de l'Éducation Nationale du Préscolaire et des Sports

Série ou Option : ....

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NOTATION PARTIELLE

So F is bijective from $[0;1]$ to $F([0;1]) = [F(0);F(1)] = [0;\beta]$

$F(0) = \int_0^0 e^{t^2}dt = 0$, $\cancel{F(1)} = \int_0^1 e^{t^2}dt = \beta$

0,75

2-a We have F is continuous on $[0;1]$ Thus F^{-1} is continuous on $[0;\beta]$. We have $\int_0^\beta F^{-1}(\cancel{t})d\cancel{t} = \lim\limits_{n\to+\infty} \frac{\beta}{n} \sum\limits_{k=1}^{k=n} F^{-1}\left(\frac{k}{n}\beta\right)$

Riman Sequence

Thus : $\lim\limits_{n\to+\infty} \frac{1}{n} \sum\limits_{k=1}^{k=n} F^{-1}\left(\frac{k}{n}\beta\right) = \frac{1}{\beta}\int_0^\beta F^{-1}(t)dt$

We devide by β $(\beta \in \mathbb{R})$.

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Thus (S_n) is convergent to l and $l = \frac{1}{\beta}\int_0^\beta F^{-1}(t)dt$.

2-b We denote $u = F^{-1}(t)$ $\cancel{\Leftrightarrow du = dt\,(F^{-1}(t))'}$

$\cancel{du = dt\,\frac{1}{F'(F}}$

$\Leftrightarrow$ $F(u) = t$

$\Leftrightarrow$ $F'(u)\,du = dt$ and When $t \to 0$ $u \to 0$

$\Leftrightarrow (q=1)$ $e^{u^2}du = dt$ because $F(0) = 0$

$t \to \beta$ $u \to$

because $F(1) = \beta$

Thus $l = \frac{1}{\beta}\int_0^1 u e^{u^2}du$.

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2-c We have $l = \frac{1}{\beta}\int_0^1 u e^{u^2}du = \frac{1}{\beta}\int_0^1 \frac{2u}{2} e^{u^2}du$

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N.B : Il est interdit aux candidats de signer leur composition ou d'y mettre un signe quelconque pouvant révéler leur identité

NOTATION PARTIELLE

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Thus $I = \frac{1}{2B} \int_0^1 (e^{u^2})' du$

$I = \frac{1}{2B} [e^{u^2}]_0^1$

Thus $\boxed{I = \frac{e-1}{2B}}$

Exercise 3:

Part I:

1-a $\Delta = (-2i)^2 - 4q \quad (i^2 = -1)$

$\Rightarrow \Delta = -4(1+q)$

1-b if $\Delta \neq 0$ So (E_q) admits two different solutions

$\Delta \neq 0 \Rightarrow 1+q \neq 0 \Rightarrow q \neq -1$

So $q \in \mathbb{C} - \{-1\}$

2 $z_1 + z_2 = \frac{+2i}{1} = 2i$ and $z_1 \cdot z_2 = \frac{q}{1}$

Part II:

1-a) if $q \neq -1 \Rightarrow m^2 - 2m \neq -1$

$\Rightarrow$ ~~$m(m-2$~~ $\quad m^2 - 2m + 1 \neq 0$

$(m-1)^2 \neq 0$

$\Rightarrow m \neq 1$

So We have z_1 and z_2

We have $\Delta = -4(1+q) = -4(m^2 - 2m + 1)$

$= -4(m-1)^2 = (2i(m-1))^2$

Thus $\delta = 2i(m-1)$ or $\delta = -2i(m-1)$

So $z_1 = \frac{2i(m-1) + 2i}{2} \Rightarrow z_1 = i(m-1) + i$

$z_1 = im$

We have $z_1 + z_2 = 2i \Rightarrow z_2 = 2i - z_1$

$\Rightarrow z_2 = 2i - im$

النقطة الجزئية

1-b We have $\frac{z_1 - 0}{z_2 - 0} = \frac{z_1}{z_2} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

and $\frac{2-m}{m} \in \mathbb{R} \,/\, m \neq 0$ because $m \in \mathbb{R}$

Thus O, M_1 and M_2 are colinear.

(0,75)

2-a $\frac{z_1}{z_2} \in i\mathbb{R} \Leftrightarrow Re\left(\frac{z_1}{z_2}\right) = 0 \Leftrightarrow Re\left(z_1 \cdot \frac{1}{z_2}\right) = 0$

$\Leftrightarrow Re\left(\frac{z_1 \cdot \overline{z_2}}{|z_2|}\right) = 0$

$|z_2| \in \mathbb{R}$ $\Leftrightarrow \frac{1}{|z_2|} Re(z_1 \cdot \overline{z_2}) = 0$

$\Leftrightarrow Re(z_1 \cdot \overline{z_2}) = 0$

(0,25)

2-b We have $|z_1 - z_2|^2 = (z_1 - z_2)(\overline{z_1 - z_2})$

$= (z_1 - z_2)(\overline{z_1} - \overline{z_2})$

① $|z_1 - z_2|^2 = z_1 \cdot \overline{z_1} - \overline{z_2} z_1 - z_2 \cdot \overline{z_1} + z_2 \cdot \overline{z_2}$

$\cancel{= |z_1|^2 + |z_2|^2 + 2 \cdot 1}$

$\cancel{= z_1 \cdot \overline{z_1} + z_2 \cdot \overline{z_2} + 2 \overline{z_2} \cdot z_1}$

(0,5)

and $|z_1 + z_2|^2 - 4Re(z_1 \cdot \overline{z_2}) = (z_1 + z_2)(\overline{z_1} + \overline{z_2}) - 4Re(z_1 \cdot \overline{z_2})$

$= z_1 \cdot \overline{z_1} + z_2 \overline{z_1} + z_1 \overline{z_2} + z_2 \cdot \overline{z_2} - 4Re(z_1 \cdot \overline{z_2})$

($2Re(z_1 \cdot \overline{z_2}) = z_2 \cdot \overline{z_1} + \overline{z_2} \cdot z_1$ Property)

$\Rightarrow \quad = z_1 \cdot \overline{z_1} + z_2 \cdot \overline{z_2} + z_2 + z_2 \cdot \overline{z_1} + z_1 \cdot \overline{z_2} - 2(\overline{z_1} z_2 + \overline{z_2} \cdot z_1)$

$|z_1 + z_2|^2 - 4Re(z_1 \cdot \overline{z_2}) = z_1 \cdot \overline{z_1} - \overline{z_1} z_2 - \overline{z_2} z_1 + \overline{z_2} \cdot z_2$ ②

Thus ① and ②: $|z_1 - z_2|^2 = |z_1 + z_2|^2 - 4Re(z_1 \cdot \overline{z_2})$

2-c We have by q 2-a $\frac{z_1}{z_2} \in i\mathbb{R} \Leftrightarrow Re(z_1 \cdot \overline{z_2}) = 0$

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مجموع نقط الصفحة

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المملكة المغربية
ⵜⴰⴳⵍⴷⵉⵜ ⵏ ⵍⵎⵖⵔⵉⴱ
وزارة التربية الوطنية والتعليم الأولي والرياضة
ⵜⴰⵎⴰⵡⴰⵙⵜ ⵏ ⵜⵎⵓⵔⵉ ⵜⴰⵏⴰⵎⵓⵔⵜ ⴷ ⵜⴰⵔⴰⵢⵔⴰ ⵜⴰⵣⵡⴰⵔⵓⵜ

امتحان نيل شهادة البكالوريا

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الشعبة أو المسلك : ....

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رقم الأرشفة

Thus According to q.2-b $|z_1-z_2|^2=|z_1+z_2|^2$

$\Leftrightarrow |z_1-z_2|=|z_1+z_2|$

$\Leftrightarrow |z_1-z_2|=|2-im+im|$

$\Leftrightarrow |z_1-z_2|=2.$

3-a) We have $(z_1-z_2)^2=(im-2i+im)^2$

$=(2im-2i)^2$

$=4i^2(m-1)^2=4i^2(m^2-2m+1)$

$=\cancel{4((im)^2-2im+1)}$

$=\cancel{4(-m^2-2im+1)}$

$=\cancel{4(}$

$=4i^2\left(m-\frac{1}{i}\right)^2$

$=\cancel{-4(m+i)^2}$

$=\cancel{-4(m^2}$

$=4i^2(1+q)$

$=-4(1+q)=\Delta$

النقطة الجزئية

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مجموع نقط الصفحة

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تنبيه : يمنع على المترشح(ة) الإمضاء أو وضع أي علامة يمكنها كشف هويته(ا)

Royaume du Maroc

Ministère de l'Éducation Nationale du Préscolaire et des Sports

EXAMEN D'OBTENTION DU CERTIFICAT DU BACCALAUREAT

Série ou Option : ....

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NOTATION PARTIELLE

Exercise 5:

1-a) We have p and q are prime Numbers and $x \wedge p = 1$ and $x \wedge q = 1$

So According to fermat's theorem $\begin{cases} x^{p-1} \equiv 1 [p] \\ x^{q-1} \equiv 1 [q] \end{cases}$

$\Leftrightarrow \begin{cases} x^{p-1} - 1 \equiv 0 [p] \quad (1) \\ x^{q-1} - 1 \equiv 0 [q] \quad (2) \end{cases}$

Thus: from (1) $x^{p-1} - 1$ is divisible by p

from (2) $x^{q-1} - 1$ is divisible by q

1

1-b We have $p \mid x^{p-1} - 1 \Leftrightarrow x^{p-1} \equiv 1 [p]$

$\Leftrightarrow x^{(p-1)(q-1)} \equiv 1 [p]$

$\Leftrightarrow x^{(p-1)(q-1)} - 1 \equiv 0 [p] \quad (1)$ $\left(a \equiv b [p] \Leftrightarrow a^n \equiv b^n [p] \right)$

$q \mid x^{q-1} - 1 \Leftrightarrow x^{q-1} - 1 \equiv 0 [q]$

$\Leftrightarrow x^{q-1} \equiv 1 [q]$ $n \in \mathbb{N}$

$\Leftrightarrow x^{(q-1)(p-1)} \equiv 1 [q]$

$\Leftrightarrow x^{(q-1)(p-1)} - 1 \equiv 0 [q] \quad (2)$

Thus from (1) $\begin{cases} p \mid x^{(p-1)(q-1)} - 1 \\ q \mid x^{(p-1)(q-1)} - 1 \end{cases}$

0,25

1-b We have p is prime and q is prime

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N.B : Il est interdit aux candidats de signer leur composition ou d'y mettre un signe quelconque pouvant révéler leur identité

NOTATION PARTIELLE

Thus $p \wedge q = 1$ and $p \mid x^{(p-1)(q-1)} - 1$ and $q \mid x^{(p-1)(q-1)} - 1$

Thus $p.q \mid x^{(p-1)(q-1)} - 1$

2 We have 13 is prime and 17 is prime and $17 \nmid 2024$ and $13 \nmid 2024$

$(2024) = 2^3 \cdot 253$

So According to the previous question

$13 \times 17 \mid 2024^{(13-1)(17-1)} - 1$

$\Rightarrow \quad 2024^{192} \equiv 1 \, [221]$

$\Rightarrow \quad 2024^{192} x = x \, [221]$

Thus $\quad 3 + 221K = x + 221K'$

$\Rightarrow \quad x = 3 + 221(\underbrace{K - K'}_{\in \mathbb{Z}})$

We put $n = K - K' / n \in \mathbb{Z}$

So $S = \{ x = 3 + 221n \,/\, n \in \mathbb{Z} \}$

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Exercise 4:

1-a) $(i;2)T(1;i) = (i\bar{i} + 1; 2i)$

$= (-i^2 + 1; 2i)$

$(i;2)T(1;i) = (2; 2i)$

$(1;i)T(i;2) = (1 \cdot 2 + i; 2i)$

$(1;i)T(i;2) = (2 + i; 2i)$

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1-b We have $((1;i),(i;2)) \in (\mathbb{C} \times \mathbb{C}^*)^2$

and According to 1a $(i;2)T(1;i) \neq (1;i)T(i;2)$

Thus the law T is not commutative in $\mathbb{C} \times \mathbb{C}^*$

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N.B : Il est interdit aux candidats de signer leur composition ou d'y mettre un signe quelconque pouvant révéler leur identité

النقطة الجزئية

2. We denote $\forall\big((a;b);(c;d);(x;y)\big)\in(\mathbb{C}\times\mathbb{C}^*)^3$

We have $\big((a;b)T(c;d)\big)T(x;y)=(a\bar{d}+c;\ bd)T(x;y)$

$=\big((a\bar{d}+c)\bar{y}+x;\ bdy\big)$ (1)

and $(a;b)T\big((c;d)T(x;y)\big)=(a;b)T(c\bar{y}+x;\ dy)$

$=(a\overline{dy}+c\bar{y}+x;\ bdy)$

$=\big(\bar{y}(a\bar{d}+c)+x;\ bdy\big)$ (2)

From (1) and (2) We deduce That T is associative.

3. We have $\forall(a;b)\in\mathbb{C}\times\mathbb{C}^*$

$(a;b)T(0;1)=(a\bar{1}+0;\ b1)$

$=(a;b)$

and $(0;1)T(a;b)=(0\bar{b}+a;\ b\times 1)$

$=(a;b)$

So yes the neutral element of T in $\mathbb{C}\times\mathbb{C}^*$ is $(0;1)$.

4-a) We have $\forall(a;b)\in\mathbb{C}\times\mathbb{C}^*$ $\quad \frac{-a}{b}\in\mathbb{C}\ /\ b\neq 0$

$\frac{1}{b}\in\mathbb{C}^*\ /\ b\neq 0$

Thus $(a;b)T\left(\frac{-a}{b};\frac{1}{b}\right)=\left(a\times\overline{\left(\frac{1}{b}\right)}-\frac{a}{b};\ b\times\frac{1}{b}\right)$

$=\left(\frac{a}{b}-\frac{a}{b};\ 1\right)$

$=(0;1)$

Thus: $\forall(a;b)\in\mathbb{C}\times\mathbb{C}^*\quad (a;b)T\left(\frac{-a}{b};\frac{1}{b}\right)=(0;1)$

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مجموع نقط الصفحة

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المملكة المغربية
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النقطة الجزئية

4-b) * We have T is Associative in $\mathbb{C}\times\mathbb{C}^*$ (q-1-b)

* We have $(0;1)$ is the neutral element in $\mathbb{C}\times\mathbb{C}^*$, q-3

* And According to 4-a $\forall (a;b)\in \mathbb{C}\times\mathbb{C}^*$

$(\frac{-a}{b};\frac{1}{b})\in \mathbb{C}\times\mathbb{C}^*$ because of : $a\in\mathbb{C}$ and $b\in\mathbb{C}^*$

the symetry of $(a;b)$ With respect To the law T $\Rightarrow$ $\frac{-a}{b}\in\mathbb{C}$ and $\frac{1}{b}\in\mathbb{C}^*$

* and in (q-1-b) T is not commutative in $\mathbb{C}\times\mathbb{C}^*$

Thus $(\mathbb{C}\times\mathbb{C}^*; T)$ is a non commutative group.

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3-a) We have $\forall ((a;b);(c;d))\in(\mathbb{R}\times\mathbb{R}^*)^2$

$(a;b)T(c;d) = (a\bar{d}+c;bd)$

We have $d\in\mathbb{R}^*$ so $\bar{d}=d$

Thus $ad+c\in\mathbb{R}$ and $bd\in\mathbb{R}$ because

$((a;b);(c;d))\in(\mathbb{R}\times\mathbb{R}^*)^2$

Thus : $\forall((a;b);(c;d))\in(\mathbb{R}\times\mathbb{R}^*)^2$

$(a;b)T(c;d)\in(\mathbb{R}\times\mathbb{R}^*)^2$

Thus : $\mathbb{R}\times\mathbb{R}^*$ is stable for the law T

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تنبيه : يمنع على المترشح(ة) الإمضاء أو وضع أي علامة يمكنها كشف هويته(ا)

مجموع نقط الصفحة

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Royaume du Maroc

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NOTATION PARTIELLE

3-b We have i-q Ī-a $\mathbb{R}\times\mathbb{R}^*$ is a stable for the law T

We have $(\mathbb{R}\times\mathbb{R}^*) \subset (\mathbb{C}\times\mathbb{C}^*)$

and $\forall (a;b) \in (\mathbb{R}\times\mathbb{R}^*)$ $\left(\frac{-a}{\bar{b}};\frac{1}{b}\right) \in (\mathbb{R}\times\mathbb{R}^*)$

because $a \in \mathbb{R}$ and $b \in \mathbb{R}^* \Rightarrow \bar{b}=b$ and $-\frac{a}{b} \in \mathbb{R}$ $b\neq 0$

and $\frac{1}{b} \in \mathbb{R}$ $b \neq 0$

Thus $\mathbb{R}\times\mathbb{R}^*$ is a subgroup of the group $(\mathbb{C}\times\mathbb{C}^*;T)$.

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last question Complex Exercise 3:

3-b OM_1M_2 is rectangular at O $\Leftrightarrow (\overrightarrow{OM_2};\overrightarrow{OM_1}) = \frac{\pi}{2}[\pi]$

$\Leftrightarrow Arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}[\pi] \Leftrightarrow \frac{z_1}{z_2} \in i\mathbb{R}$

$\Leftrightarrow |z_1 - z_2| = 2$ (of previous questions)

We have $(z_1-z_2)^2 = \Delta \Leftrightarrow |z_1-z_2|^2 = |\Delta|$

$\Leftrightarrow 4 = |-4(\lambda+9)|$

$\Leftrightarrow |\lambda+9| = 1$

So the set of points Ω is Γ is a circle of center A(-1) and radius 1.

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